Divergence Test: If $\lim_{n\to\infty} a_n$ is not zero or does not exist, then the series $\sum_{n=1}^{\infty} a_n$ ________________.

Integral Test: Suppose f is a ________________, ________________, ________________ function on $[1,\infty)$ and let $a_n = f(n)$. Then the series $\sum_{n=1}^{\infty} a_n$ is convergent if and only if the improper integral $\int_1^{\infty} f(x)\,dx$ is ________________.

Comparison Test: Suppose that $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are series with ________________ terms. If $\sum_{n=1}^{\infty} b_n$ is convergent and $a_n \le b_n$ for all n, then $\sum_{n=1}^{\infty} a_n$ is also ________________. If $\sum_{n=1}^{\infty} b_n$ is divergent and $a_n \ge b_n$ for all n, then $\sum_{n=1}^{\infty} a_n$ is also ________________.

Limit Comparison Test: Suppose that $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are series with ________________ terms. If $\lim_{n\to\infty} \frac{a_n}{b_n} = c$ where c is ________________ and ________________, then either both series converge or both diverge.

Alternating Series Test: Suppose that we have an alternating series. That is $a_n = (-1)^n\,|a_n|$ or $a_n = (-1)^{n-1}\,|a_n|$ (assume $a_n > 0$). If ________________ for all n and ________________, then the series $\sum_{n=1}^{\infty} a_n$ is ________________.

Ratio Test: Let $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L$.

(a) If $L < 1$, then ____.

(b) If $L > 1$, or $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \infty$, then ____.

(c) If $L = 1$, then ____.

Determine if the following series are absolutely convergent, conditionally convergent, or divergent.

1. $\displaystyle\sum_{n=1}^{\infty} \big(\arctan(n+1) - \arctan(n) \big)$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^{3n}}{3^{n-1}}$

3. $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{1}{n\big(\ln(n)\big)^2}$

4. $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n}{2^n}$

5. $\displaystyle\sum_{n=3}^{\infty} \cos(n\pi) n^{-4}$

6. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{3n+7}$

7. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}n}{5n+6}$

8. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(n+2)!}$

9. $\displaystyle\sum_{n=2}^{\infty} \big(\cos(n)\big)^3 (n^3 - n)^{-1}$

10. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt[3]{n-1}}$